# A MATLAB Script for Predicting Lunar Eclipses

This document describes a MATLAB script named `leclipse.m` that can be used to predict local circumstances of lunar eclipses. This software provides the eclipse type, the universal times and topocentric coordinates of the Moon at the beginning and end of the penumbra contacts, and the time and coordinates at maximum eclipse. The source ephemeris for this routine is a JPL binary ephemeris file. This application uses several functions ported to MATLAB from the Fortran version of the NOVAS (Naval Observatory Vector Astrometry Subroutines) source code developed at the United States Naval Observatory (www.usno.navy.mil/USNO/astronomical-applications/software-products/novas). JPL binary ephemeris files for Windows compatible computers can be downloaded at www.cdeagle.com.

This MATLAB script uses a combination of one-dimensional minimization and root-finding to solve this classic problem. The objective function used in these calculations involves the geocentric separation angle between the center of the Moon and the anti-Sun position vector or shadow axis, and the semidiameter and horizontal parallax of the Sun and Moon. This function is given by the following expression:

$$f(t) = \cos^{-1}\left(-\hat{\mathbf{u}}_m \bullet \hat{\mathbf{u}}_s\right) - f_1 + s_m$$

where

$\hat{\mathbf{u}}_s =$ geocentric unit position vector of the Sun

$\hat{\mathbf{u}}_m =$ geocentric unit position vector of the Moon

$f_1 = 1.02\left(\pi_1 + \pi_s + s_s\right) =$ size of penumbra shadow

$\pi_1 = 0.99834\pi_m =$ corrected parallax

$\pi_m =$ horizontal parallax of the Moon

$\pi_s =$ horizontal parallax of the Sun

$s_m =$ semidiameter of the Moon

$s_s =$ semidiameter of the Sun

If we let $\sigma$ represent the minimum geocentric separation angle of the Moon relative to the shadow axis, a penumbral lunar eclipse occurs whenever the following geometric condition is satisfied:

$$\sigma < 1.02\left(\pi_1 + \pi_s + s_s\right) + s_m$$

A partial lunar eclipse will happen whenever the following is true:

$$\sigma < 1.02\left(\pi_1 + \pi_s - s_s\right) + s_m$$

The geometric condition for a total lunar eclipse is given by

$$\sigma < 1.02\left(\pi_1 + \pi_s - s_s\right) - s_m$$

In these expressions

$$\pi_m = \sin^{-1}\left(\frac{r_{eq}}{d_m}\right) \qquad \pi_s = \sin^{-1}\left(\frac{r_{eq}}{d_s}\right)$$

$$s_m = \sin^{-1}\left(\frac{r_m}{d_m}\right) \qquad s_s = \sin^{-1}\left(\frac{r_s}{d_s}\right)$$

where $r_{eq}$ is the equatorial radius of the Earth (6378.14 kilometers), $r_m$ is the radius of the Moon (1738 kilometers), $r_s$ is the radius of the Sun (696,000 kilometers), $d_m$ is the geocentric distance of the Moon, and $d_s$ is the geocentric distance of the Sun.

The following is a typical user interaction with this script. The screen output created by the script illustrates the local circumstances of a total lunar eclipse. The initial calendar date was January 1, 2000, the search duration was 30 days, and the observer was located at the Chamberlin Observatory in Denver, Colorado. The calendar date and time displayed is on the UTC time scale.

```
local circumstances of lunar eclipses
=====================================

please input an initial UTC calendar date

please input the calendar date
(1 <= month <= 12, 1 <= day <= 31, year = all digits!)
? 1,1,2000


please input the search duration (days)
? 30

please input the geographic latitude of the observer
(-90 <= degrees <= +90, 0 <= minutes <= 60, 0 <= seconds <= 60)
(north latitude is positive, south latitude is negative)
? 39,40,36

please input the geographic longitude of the observer
(0 <= degrees <= 360, 0 <= minutes <= 60, 0 <= seconds <= 60)
(east longitude is positive, west longitude is negative)
? -104,57,12

please input the altitude of the observer (meters)
(positive above sea level, negative below sea level)
? 1644


total lunar eclipse
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begin penumbral phase of lunar eclipse
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calendar date             21-Jan-2000

universal time            02:02:54.489

UTC Julian date           2451564.5854
```

```
lunar azimuth angle        +83d 04m 19.72s

lunar elevation angle      +22d 44m 02.19s

greatest eclipse conditions
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calendar date             21-Jan-2000

universal time            04:43:57.941

UTC Julian date           2451564.6972

lunar azimuth angle        +111d 30m 10.33s

lunar elevation angle      +52d 11m 23.72s

end penumbral phase of lunar eclipse
------------------------------------

calendar date             21-Jan-2000

universal time            07:24:07.313

UTC Julian date           2451564.8084

lunar azimuth angle        +184d 38m 54.13s

lunar elevation angle      +69d 27m 43.86s


event duration             +05h 21m 12.8232s
```

The following are the results for this same eclipse using the Multiyear Interactive Computer Almanac (MICA) published by the United States Naval Observatory.

```
            Total Eclipse of the Moon of 2000 Jan. 21
                         Delta T:  63.8s

                  Chamberlin Obs., Denver
         Location:  W104°57'12.0", N39°40'36.0",  1644m
           (Longitude referred to Greenwich meridian)

                                          Moon's         Position
                           UT1      Altitude  Azimuth     Angle
                        d  h   m        °         °          °
Moon enters penumbra  21 02:03.0     22.7      83.1       85.8
Moon enters umbra     21 03:01.5     33.6      91.9       79.9
Moon enters totality  21 04:04.6     45.2     102.9      238.3
Maximum Eclipse       21 04:43.5     52.1     111.4        6.7
Moon exits totality   21 05:22.4     58.6     122.2      135.2
Moon exits umbra      21 06:25.5     66.9     148.5      293.6
Moon exits penumbra   21 07:24.1     69.5     184.6      287.6

         Penumbral Duration:   5h 21.2m
            Umbral Duration:   3h 24.0m
        Duration of Totality:   1h 17.8m
                  Magnitude:   1.330
```