Rise and Set of an Earth Satellite

This document describes two MATLAB scripts that can be used to determine rise and set conditions of Earth satellites relative to a ground site on an oblate Earth. The user can specify a minimum elevation or "mask" angle for the first simulation. Both scripts propagate a satellite's orbit using Kozai's method. The first script uses a combination of one-dimensional minimization and root-finding to calculate visibility conditions.

The topocentric elevation angle of an Earth satellite can be calculated from

$$E = \sin^{-1}\left(r_{z_{topo}}\right)$$

where the components of the satellite's topocentric position vector $\mathbf{r}_{ecf}$ are determined from the following transformation matrix and the ECI position vector $\mathbf{r}_{eci}$ of the satellite:

$$\mathbf{r}_{topo} = \begin{bmatrix} \sin\phi\cos\lambda & \sin\phi\sin\lambda & -\cos\phi \\ -\sin\theta & \cos\theta & 0 \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix} \mathbf{r}_{eci}$$

In this matrix-vector equation, ϕ is the geodetic latitude of the ground site, λ is the geographic longitude of the ground site and θ is the local sidereal time.

The objective function used to calculate visibility conditions is given by the expression

$$f_{obj}(t) = -E + E_{\min}$$

where $E_{\min}$ is an *optional* minimum elevation angle constraint or "mask". Notice that this is actually a maximization problem since we are using the negative value of this function.

riseset1.m – Kozai orbit propagation

This application determines rise and set conditions of an Earth satellite using Kozai's method of orbit propagation.

The following is a typical user interaction with this script.

```
              program riseset1

   < rise and set of an Earth satellite >

         < Kozai orbit propagation >


   please input the calendar date
   (1 <= month <= 12, 1 <= day <= 31, year = all digits!)
   ? 1,1,1998
```

```
please input the universal time
(0 <= hours <= 24, 0 <= minutes <= 60, 0 <= seconds <= 60)
? 0,0,0


initial orbital elements

please input the semimajor axis (kilometers)
(semimajor axis > 0)
? 8000

please input the orbital eccentricity (non-dimensional)
(0 <= eccentricity < 1)
? 0

please input the orbital inclination (degrees)
(0 <= inclination <= 180)
? 28.5

please input the right ascension of the ascending node (degrees)
(0 <= raan <= 360)
? 100

please input the true anomaly (degrees)
(0 <= true anomaly <= 360)
? 45


ground site coordinates

please input the geographic latitude of the ground site
(-90 <= degrees <= +90, 0 <= minutes <= 60, 0 <= seconds <= 60)
(north latitude is positive, south latitude is negative)
? 40,0,0

please input the geographic longitude of the ground site
(0 <= degrees <= 360, 0 <= minutes <= 60, 0 <= seconds <= 60)
(east longitude is positive, west longitude is negative)
? -105,0,0

please input the altitude of the ground site (meters)
(positive above sea level, negative below sea level)
? 0


would you like to enforce an elevation angle constraint (y = yes, n = no)
? y

please input the minimum elevation angle constraint (degrees)
? 5


please input the simulation duration in days
? 5
```

The script will ask if you would like to create a screen display of the visibility conditions during the simulation. This prompt appears as follows:

```
would you like screen output (y = yes, n = no)
?
```

The following is a typical screen display of the first rise, maximum elevation and set conditions for this example.

```
rise conditions

calendar date            01-Jan-1998
universal time           07:44:06.765

Julian date              2450814.8223

topocentric azimuth angle     178.3280  degrees
topocentric elevation angle     5.0000  degrees

topocentric slant range      4298.6787  kilometers

maximum elevation conditions

calendar date            01-Jan-1998
universal time           07:50:01.382

Julian date              2450814.8264

topocentric azimuth angle     145.9483  degrees
topocentric elevation angle    10.2005  degrees

topocentric slant range      3829.0803  kilometers

set conditions

calendar date            01-Jan-1998
universal time           07:55:55.215

Julian date              2450814.8305

topocentric azimuth angle     113.7348  degrees
topocentric elevation angle     5.0000  degrees

topocentric slant range      4309.7357  kilometers

event duration                 11.8075  minutes
```

After the simulation is complete the software will ask if you would like to create a data file of the visibility conditions. This prompt appears as follows:

```
would you like to create an ascii data file (y = yes, n = no)
?
```

If you respond with `y` for yes, the software will ask you to input the name of the data file with

```
please input the rise-set data file name
(be sure to include a file name extension)
?
```

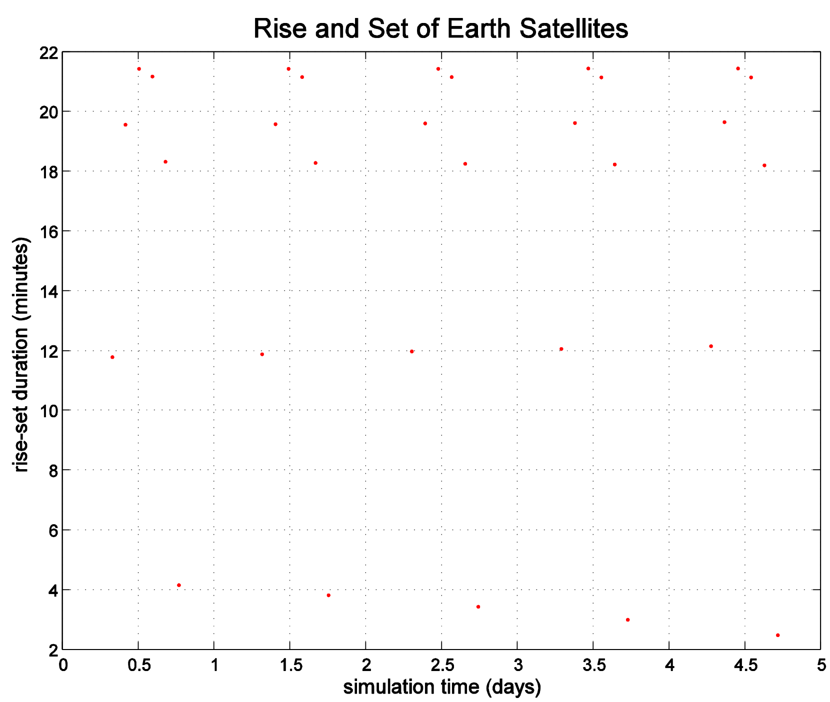
The following is part of the data file for this example. The first column is the simulation time at which visibility begins, the second column is the simulation time at maximum elevation, and the third column is the simulation time at which visibility ends. The fourth column is the event duration in minutes.

```
 rise time     max time     set time    duration
  (days)        (days)       (days)     (minutes)

   0.3223       0.3264       0.3305      11.8075
   0.4071       0.4139       0.4207      19.5760
   0.4943       0.5017       0.5092      21.4471
   0.5823       0.5896       0.5970      21.1856
```

The script will also ask if you would like to create a graphics display of the event duration versus simulation time with this final prompt. The form of this prompt is

```
would you like to display graphics (y = yes, n = no)
?
```

If you respond with `y` for yes, the software will create a graphics display similar to the following:



griseset.m – graphics display of visibility conditions

This script graphically displays rise and set conditions using Kozai's method for orbit propagation. This software simply propagates a satellite's orbit for a user specified time duration and step size while collecting rise-set information whenever the elevation angle of the Earth satellite is positive.

After the orbit propagation is complete the user can elect to graphically display one or more of the following items:

- topocentric azimuth
- topocentric elevation
- topocentric slant range
- topocentric azimuth rate
- topocentric elevation rate
- topocentric range rate

The following is a typical user interaction with this script.

```
             program griseset

< graphics display of rise-set conditions >

       < Kozai orbit propagation >

please input the calendar date
(1 <= month <= 12, 1 <= day <= 31, year = all digits!)
? 1,1,1998

please input the universal time
(0 <= hours <= 24, 0 <= minutes <= 60, 0 <= seconds <= 60)
? 0,0,0

initial orbital elements

please input the semimajor axis (kilometers)
(semimajor axis > 0)
? 8000

please input the orbital eccentricity (non-dimensional)
(0 <= eccentricity < 1)
? 0

please input the orbital inclination (degrees)
(0 <= inclination <= 180)
? 28.5

please input the right ascension of the ascending node (degrees)
(0 <= raan <= 360)
? 100

please input the true anomaly (degrees)
(0 <= true anomaly <= 360)
? 45

ground site coordinates

please input the geographic latitude of the ground site
(-90 <= degrees <= +90, 0 <= minutes <= 60, 0 <= seconds <= 60)
(north latitude is positive, south latitude is negative)
? 40,0,0
```

```
please input the geographic longitude of the ground site
(0 <= degrees <= 360, 0 <= minutes <= 60, 0 <= seconds <= 60)
(east longitude is positive, west longitude is negative)
? -105,0,0

please input the altitude of the ground site (meters)
(positive above sea level, negative below sea level)
? 0

please input the simulation duration in days
? 1

please input the step size in seconds
? 10
```

After the propagation is complete the script will ask you what data item to plot. This request and menu appear as follows:

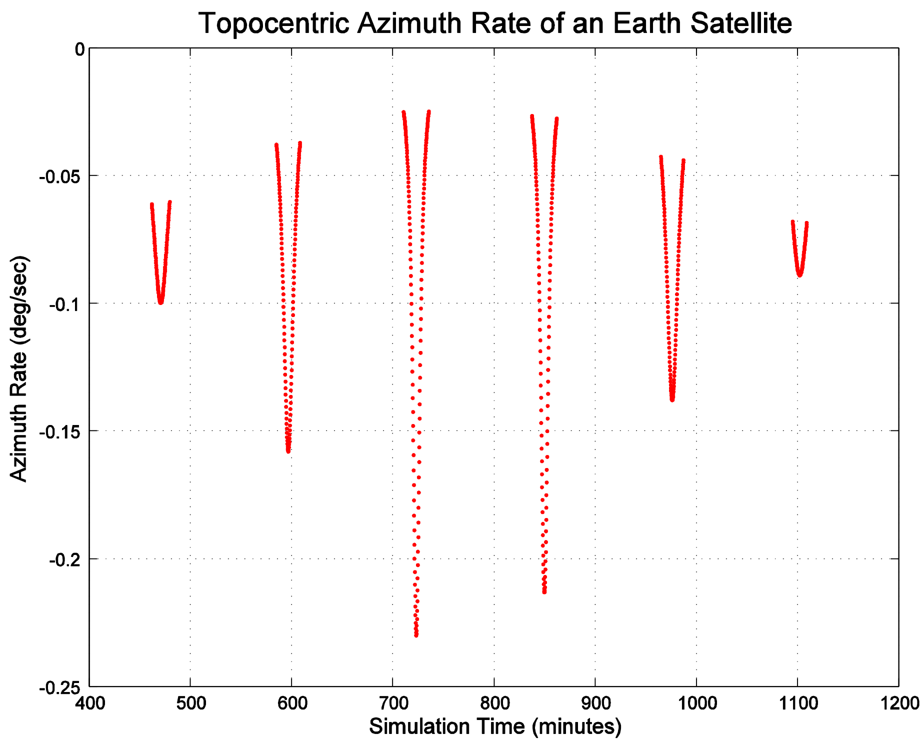
```
please select the item to plot

 <1> topocentric azimuth

 <2> topocentric elevation

 <3> topocentric slant range

 <4> topocentric azimuth rate

 <5> topocentric elevation rate

 <6> topocentric range rate

?
```

After a plot is created the user can elect to create another plot by responding with `y` for yes to the following prompt.

```
would you like to create another plot (y = yes, n = no)
?
```

If you respond with `n` for no the script will terminate.

In this application the topocentric elevation angle is corrected for atmospheric refraction. The following is a typical graphic display created with this software. The data points are plotted at the step size input by the user.



Kozai orbit propagation

According to Yoshihide Kozai's method, "The Motion of a Close Earth Satellite", *The Astronomical Journal*, **64**, No. 1274, pp. 367-377, the time evolution of the mean orbital elements due to first-order secular perturbations of the gravity harmonic J_2 is as follows:

$$M(t) = M_0 + \tilde{n}(t - t_0)$$

$$\Omega(t) = \Omega_0 + \dot{\Omega}(t - t_0)$$

$$\omega(t) = \omega_0 + \dot{\omega}(t - t_0)$$

where M_0 is the mean anomaly, Ω_0 is the right ascension of the ascending node (RAAN) and ω_0 is the argument of perigee, all at the initial time t_0. In the first expression $\tilde{n}$ is called the *perturbed mean motion* and is equal to the time rate of change of mean anomaly.

The perturbed mean motion can be calculated from:

$$\tilde{n} = \frac{dM}{dt} = n\left\{1 + \frac{3}{2} J_2 \left(\frac{r_{eq}}{p}\right)^2 \sqrt{1 - e^2}\left(1 - \frac{3}{2}\sin^2 i\right)\right\}$$

where

J_2 = Earth oblateness gravity term

r_{eq} = equatorial radius of the Earth

a = semimajor axis

e = orbital eccentricity

i = orbital inclination

n = unperturbed or Keplerian mean motion $= \sqrt{\mu / a^3}$

$p = a\left(1-e^2\right)$ = semiparameter

The time rate of change of RAAN is determined from

$$\dot{\Omega} = \frac{d\Omega}{dt} = -\frac{3}{2} J_2 \tilde{n} \left(\frac{r_{eq}}{p} \right)^2 \cos i$$

The secular perturbation of the argument of perigee is given by:

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{3}{2} J_2 \tilde{n} \left(\frac{r_{eq}}{p} \right)^2 \left(2 - \frac{5}{2} \sin^2 i \right)$$

Greenwich apparent sidereal time

The MATLAB function that calculates the apparent Greenwich sidereal time using the first few terms of the IAU 1980 nutation algorithm.

The Greenwich apparent sidereal time is given by the expression

$$\theta = \theta_m + \Delta\psi \cos\left(\varepsilon_m + \Delta\varepsilon\right)$$

where θ_m is the Greenwich mean sidereal time, $\Delta\psi$ is the nutation in longitude, ε_m is the mean obliquity of the ecliptic and $\Delta\varepsilon$ is the nutation in obliquity.

The Greenwich <u>mean</u> sidereal time is calculated using the expression

$$\theta_m = 280.46061837 + 360.98564736629\left(JD - 2451545.0\right) + 0.000387933T^2 - T^3 / 38710000$$

where $T = (JD - 2451545)/36525$ and JD is the Julian date. The mean obliquity of the ecliptic is determined from

$$\varepsilon_m = 23^0 26' 21.'' 448 - 46.'' 8150T - 0.'' 00059T^2 + 0.'' 001813T^3$$

The nutation in obliquity and longitude involve the following three trigonometric arguments (in degrees):

$$L = 280.4665 + 36000.7698T$$

$$L' = 218.3165 + 481267.8813T$$

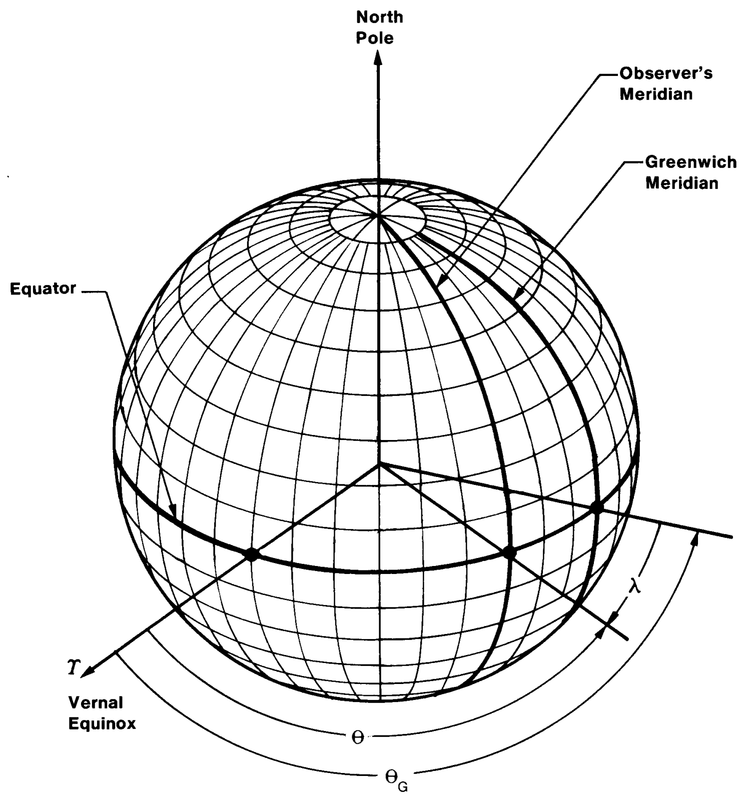
$$\Omega = 125.04452 - 1934.136261T$$

The calculation of nutation uses the following two equations:

$$\Delta\psi = -17.20\sin\Omega - 1.32\sin 2L - 0.23\sin 2L' + 0.21\sin 2\Omega$$

$$\Delta\varepsilon = 9.20\cos\Omega + 0.57\cos 2L + 0.10\cos 2L' - 0.09\cos 2\Omega$$

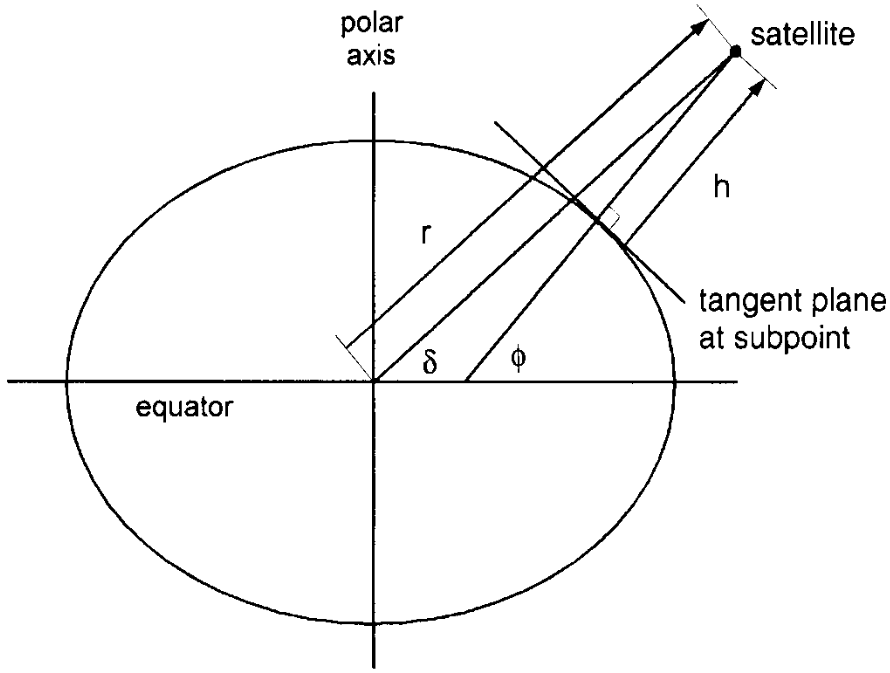
These corrections are in units of arc seconds. The following diagram illustrates the geometry of sidereal time.



Sidereal Time

Geodetic coordinates

These MATLAB scripts use a series solution to convert geocentric distance and declination to geodetic altitude and latitude. The following diagram illustrates the geometric relationship between geocentric and geodetic coordinates.



In this diagram, δ is the geocentric declination, ϕ is the geodetic latitude, r is the geocentric distance, and h is the geodetic altitude. The exact mathematical relationship between geocentric and geodetic coordinates is given by the following system of two nonlinear equations

$$(c+h)\cos\phi - r\cos\delta = 0$$

$$(s+h)\sin\phi - r\sin\delta = 0$$

where the geodetic constants c and s are given by

$$c = \frac{r_{eq}}{\sqrt{1-\left(2f-f^2\right)\sin^2\phi}} \qquad s = c\left(1-f\right)^2$$

and r_{eq} is the Earth equatorial radius (6378.14 kilometers) and f is the flattening factor for the Earth (1/298.257).

The geodetic latitude is determined using the following expression:

$$\phi = \delta + \left(\frac{\sin 2\delta}{\rho}\right) f + \left[\left(\frac{1}{\rho^2} - \frac{1}{4\rho}\right)\sin 4\delta\right] f^2$$

The geodetic altitude is calculated from

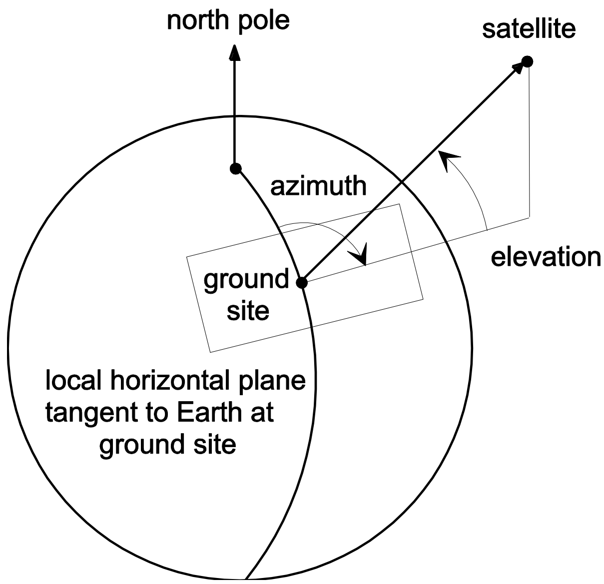
$$\hat{h} = (\hat{r}-1) + \left\{\left(\frac{1-\cos 2\delta}{2}\right) f + \left[\left(\frac{1}{4\rho} - \frac{1}{16}\right)(1-\cos 4\delta)\right] f^2\right\}$$

In these equations, ρ is the geocentric distance of the satellite, $\hat{h} = h / r_{eq}$ and $\hat{r} = \rho / r_{eq}$.

Topocentric coordinates

These MATLAB scripts calculate the topocentric coordinates (azimuth, elevation and slant range) and their rates with respect to an observer or ground site on an oblate Earth.

The following diagram illustrates the geometry of topocentric coordinates. Azimuth is measured positive clockwise from north and elevation is positive above the local horizontal plane.



The transformation of an ECI position vector $\mathbf{r}_{eci}$ to a topocentric position vector $\mathbf{r}_{topo}$ is given by the following vector-matrix operation

$$\mathbf{r}_{topo} = \left[\mathbf{T}\right]\mathbf{r}_{eci} = \begin{bmatrix} \sin\phi\cos\theta & \sin\phi\sin\theta & -\cos\phi \\ -\sin\theta & \cos\theta & 0 \\ \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \end{bmatrix}\mathbf{r}_{eci}$$

where ϕ is the geodetic latitude of the ground site and θ is the local sidereal time at the ground site. The local sidereal time of a ground site is given by

$$\theta = \theta_{g0} + \omega_e t + \lambda_e$$

where θ_{g0} is the Greenwich sidereal time at 0 hours universal time, ω_e is the inertial rotation rate of the Earth, *t* is the elapsed time since 0 hours universal time and λ_e is the east longitude of the ground site.

The ECI position vector used in this transformation is the position of the satellite relative to the observer or ground site. It is determined from the ECI position vectors of the observer $\mathbf{r}_{obs}$ and satellite $\mathbf{r}_{sat}$ according to

$$\mathbf{r}_{eci} = \mathbf{r}_{sat} - \mathbf{r}_{obs}$$

The scalar slant range from the observer to the satellite is computed from the components of this vector according to

$$p = \sqrt{x_{eci}^2 + y_{eci}^2 + z_{eci}^2}$$

The topocentric azimuth angle is calculated from the *x* and *y* components of the topocentric position vector using the following expression

$$A = \tan^{-1}\left(r_{y_{topo}}, -r_{x_{topo}}\right)$$

The topocentric elevation angle is calculated from the *z* component of the topocentric *unit* position vector with this next expression

$$E = \sin^{-1}\left(r_{z_{topo}}\right)$$

Azimuth is measured positive clockwise from north (90° is east, 180° is south, etc.) and elevation is positive above the local horizontal or tangent plane at the observer's geographic location or ground site.

The ECI range-rate vector $\dot{\mathbf{p}}_{eci}$ of the satellite relative to the observer is determined from

$$\dot{\mathbf{p}}_{eci} = \mathbf{v}_{sat} - \mathbf{w} \times \mathbf{r}_{sat}$$

where $\mathbf{w} = \omega_e \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ is the inertial rotation vector of the Earth and $\mathbf{v}_{sat}$ is the ECI velocity vector of the satellite.

The topocentric range-rate vector is computed from the transformation

$$\dot{\mathbf{p}}_{topo} = \left[\mathbf{T}\right]\dot{\mathbf{p}}_{eci}$$

The derivative of slant range or range-rate is given by

$$\dot{p} = \frac{\mathbf{r}_{topo} \bullet \dot{\mathbf{p}}_{topo}}{p}$$

The azimuth and elevation rates are determined from the *x*, *y* and *z* components of the topocentric range and range-rate vectors as follows

$$\dot{A} = \frac{\dot{p}_{topo_x} p_{topo_y} - \dot{p}_{topo_y} p_{topo_x}}{p_x^2 + p_y^2}$$

$$\dot{E} = \frac{\dot{p}_{topo_z} - \dot{p}\sin E}{\sqrt{p_x^2 + p_y^2}}$$

Atmospheric refraction

The `griseset.m` MATLAB script corrects a topocentric elevation angle for atmospheric refraction. The simple correction to be added to the calculated topocentric elevation angle is as follows:

$$\Delta E = \frac{1.02}{\tan\left(E + \frac{10.3}{E + 5.11}\right)}$$

where E is the true topocentric elevation angle in degrees.