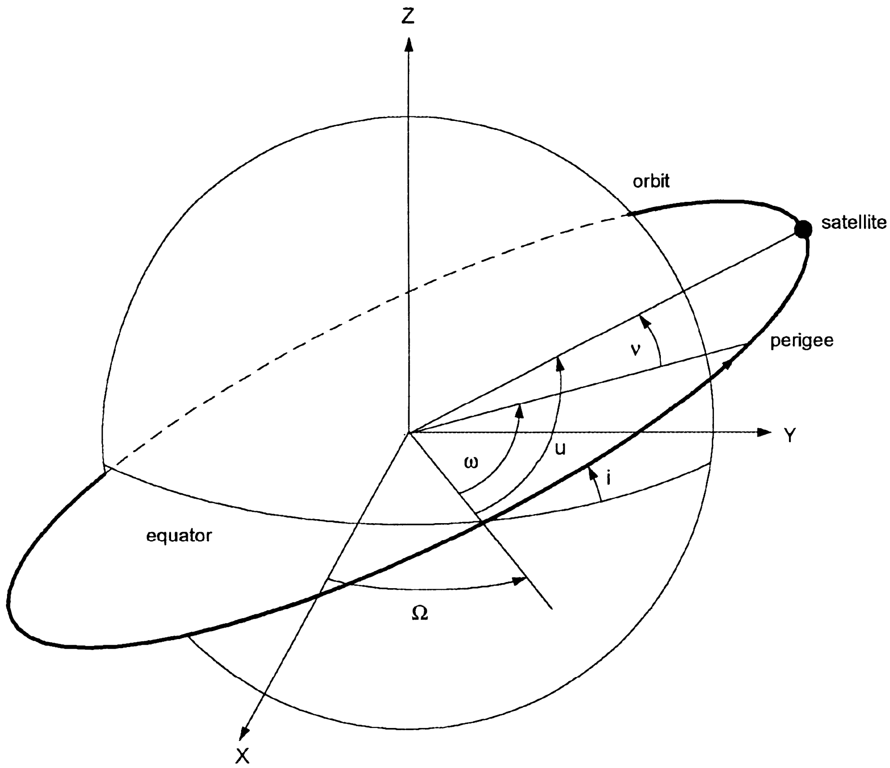
CLASSICAL ORBITAL ELEMENTS

This section describes MATLAB functions that can be used to convert between inertial state vectors and classical orbital elements. The following diagram illustrates the geometry of these orbital elements.



where

a = semimajor axis

e = orbital eccentricity

i = orbital inclination

ω = argument of perigee

Ω = right ascension of the ascending node

θ = true anomaly

The semimajor axis defines the size of the orbit and the orbital eccentricity defines the shape of the orbit. The angular orbital elements are defined with respect to a fundamental x-axis, the vernal equinox, and a fundamental plane, the equator. The z-axis of this system is collinear with the spin axis of the Earth, and the y-axis completes a right-handed coordinate system.

The orbital inclination is the angle between the equatorial plane and the orbit plane. Satellite orbits with inclinations between 0 and 90 degrees are called *direct* orbits and satellites with inclinations greater than 90 and less than 180 degrees are called *retrograde* orbits. The right ascension of the ascending node (RAAN) is the angle measured from the x-axis (vernal equinox) eastward along the equator to the ascending node. The argument is the angle from the ascending node, measured along the orbit plane in the direction of increasing true anomaly, to the argument of perigee. The true anomaly is the angle from the argument of perigee, measured along the orbit plane in the direction of motion, to the satellite's location. Also shown is the argument of latitude, *u*, which is the angle from the ascending node, measured in the orbit plane, to the satellite's location in the orbit. It is equal to $u = \nu + \omega$.

The orbital eccentricity is an indication of the type of orbit. For values of $0 \le e < 1$, the orbit is circular or elliptic. The orbit is parabolic when $e = 1$ and the orbit is hyperbolic if $e > 1$

The semimajor axis is calculated using the following expression:

$$a = \frac{1}{\dfrac{2}{r} - \dfrac{v^2}{\mu}}$$

where $r = |\mathbf{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2}$ and $v = |\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$. The angular orbital elements are calculated from modified equinoctial orbital elements which are calculated from the ECI position and velocity vectors.

The scalar orbital eccentricity is determined from *h* and *k* as follows:

$$e = \sqrt{h^2 + k^2}$$

The orbital inclination is determined from *p* and *q* using the following expression

$$i = 2\tan^{-1}\left(\sqrt{p^2 + q^2}\right)$$

For values of inclination greater than a small value ε, the right ascension of the ascending node (RAAN) is given by

$$\Omega = \tan^{-1}(p, q)$$

Otherwise, the orbit is equatorial and there is no RAAN. If the value of orbital eccentricity is greater than ε, the argument of perigee is determined from

$$\omega = \tan^{-1}(h, k) - \Omega$$

Otherwise, the orbit is circular and there is no argument of perigee. In the MATLAB code for these calculations, $\varepsilon = 10^{-8}$.