A MATLAB Script for Predicting Solar Eclipses

This document describes a MATLAB script named `seclipse.m` that can be used to predict local circumstances of solar eclipses. This computer program provides the universal times and topocentric coordinates of the Sun and Moon at the beginning and end of the penumbra contacts, and the time and coordinates at maximum eclipse. The source ephemeris for this routine is a JPL binary ephemeris file. This application uses several functions ported to MATLAB from the Fortran version of the NOVAS (Naval Observatory Vector Astrometry Subroutines) source code developed at the United States Naval Observatory (www.usno.navy.mil/USNO/astronomical-applications/software-products/novas). JPL binary ephemeris files for Windows compatible computers can be downloaded at www.cdeagle.com.

This script uses a combination of one-dimensional minimization and root-finding to solve this classical astronomy problem. The objective function used in these calculations is the difference between the selenocentric (Moon-centered) separation angle between the axis of the lunar shadow and an Earth observer, and the penumbra shadow angle. This function is given by the following expression:

$$f(t) = \cos^{-1}\left(\hat{\mathbf{u}}_{axis} \bullet \hat{\mathbf{u}}_{m-o}\right) - \psi_p$$

where

$$\hat{\mathbf{u}}_{axis} = \text{ selenocentric unit vector of the Moon's shadow}$$

$$\hat{\mathbf{u}}_{m-o} = \text{ selenocentric unit position vector of the observer}$$

$$\psi_p = \text{ penumbra shadow angle}$$

The penumbra shadow angle at the distance of the Earth observer is determined from

$$\psi_p = \sin^{-1}\left(\frac{r_m}{d_m}\right) + \sin^{-1}\left(\frac{r_s + r_m}{d_{m-s}}\right)$$

In this expression r_m is the radius of the Moon, r_s is the radius of the Sun, d_m is the topocentric distance of the Moon, and d_{m-s} is the distance from the Moon to the Sun.

The selenocentric position vector of the Sun is computed from the expression

$$\mathbf{r}_{m-s} = \mathbf{r}_s - \mathbf{r}_m$$

where $\mathbf{r}_s$ is the geocentric position vector of the Sun and $\mathbf{r}_m$ is the geocentric position vector of the Moon. The distance d_{m-s} in the equation above is the scalar magnitude of this vector.

The following is a typical user interaction with this MATLAB script. The screen output created by the script illustrates the local circumstances of a partial solar eclipse. The initial calendar date was December 25, 2000, the search duration was 30 days, and the observer was located at the Chamberlin Observatory in Denver, Colorado. The calendar date and time displayed are on the UTC time scale.

```
local circumstances of solar eclipses
=====================================

please input an initial UTC calendar date

please input the calendar date
(1 <= month <= 12, 1 <= day <= 31, year = all digits!)
? 12,1,2000

please input the search duration (days)
? 30

please input the geographic latitude of the observer
(-90 <= degrees <= +90, 0 <= minutes <= 60, 0 <= seconds <= 60)
(north latitude is positive, south latitude is negative)
? 39,40,36

please input the geographic longitude of the observer
(0 <= degrees <= 360, 0 <= minutes <= 60, 0 <= seconds <= 60)
(east longitude is positive, west longitude is negative)
? -104,57,12

please input the altitude of the observer (meters)
(positive above sea level, negative below sea level)
? 1644


begin penumbral phase of solar eclipse
--------------------------------------

calendar date            25-Dec-2000

universal time           15:29:28.878

UTC Julian date          2451904.1455

solar topocentric azimuth angle     +132d 09m 24.36s

solar topocentric elevation angle   +10d 05m 17.59s

lunar topocentric azimuth angle     +132d 10m 25.51s

lunar topocentric elevation angle   +10d 36m 24.38s


greatest eclipse conditions
---------------------------

calendar date            25-Dec-2000

universal time           16:43:49.826

UTC Julian date          2451904.1971

solar topocentric azimuth angle     +146d 57m 12.48s

solar topocentric elevation angle   +19d 22m 51.65s

lunar topocentric azimuth angle     +146d 44m 37.17s

lunar topocentric elevation angle   +19d 36m 46.75s
```

```
end penumbral phase of solar eclipse
------------------------------------

calendar date             25-Dec-2000

universal time            18:06:14.692

UTC Julian date           2451904.2543

solar topocentric azimuth angle     +166d 15m 18.44s

solar topocentric elevation angle   +25d 42m 22.42s

lunar topocentric azimuth angle     +165d 41m 32.92s

lunar topocentric elevation angle   +25d 49m 17.46s


event duration                      +02h 36m 45.8133s
```

The following are the results for this same eclipse using the Multiyear Interactive Computer Almanac (MICA) published by the United States Naval Observatory.

```
                     Solar Eclipse of 2000 Dec. 25
                Sun in Partial Eclipse at this Location
                           Delta T:  64.1s

                      Chamberlin Obs., Denver
            Location:  W104°57'12.0", N39°40'36.0",  1644m
              (Longitude referred to Greenwich meridian)

                                         Sun's            Position     Vertex
                        UT1         Altitude   Azimuth      Angle        Angle
                    d  h  m   s         °          °          °            °
   Eclipse Begins    25 15:29:29.6     10.1       132.2       319.7        358.3
   Maximum Eclipse   25 16:43:36.3     19.4       146.9
   Eclipse Ends      25 18:06:14.6     25.7       166.3        65.6         77.3

             Duration:   2h 36m 45.0s
            Magnitude:   0.396
          Obscuration:  27.3%
```

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FEATURE ARTICLE

In this issue of *Celestial Computing*, we present an interactive QuickBASIC program called **SECLIPSE.BAS** which can be used to determine the *local circumstances* of solar eclipses. The software is valid for solar eclipses which occur in the time period 1600-2200 A.D. The characteristics of solar eclipses are usually determined by Bessel's method which is described in Chapter 9 of *Explanatory Supplement to the Ephemeris*. This is also the technique used by Jean Meeus in his book, *Elements of Solar Eclipses*. However, the computer algorithm implemented in SECLIPSE.BAS uses the numerical methods of minimization and root-finding which have been discussed in past issues of *Celestial Computing* to determine local circumstances.

The algorithm used in SECLIPSE.BAS first calculates the minimum *selenocentric* or moon-centered separation angle between the shadow axis and the earth observer. The penumbra and umbra angles at the observer's distance are computed next. If the observer angle is less than the shadow angle, the root-finder is used to search backward to find the time at which the observer entered the shadow. The process is repeated in the forward direction to determine the time the observer exits the lunar shadow.

Figure 1 illustrates the geometry between the penumbra shadow and the earth observer. In this figure, ψ is the selenocentric angle between the shadow axis and the penumbra shadow at the observer's selenocentric distance. The angle ϕ is the selenocentric angle between the observer and the shadow axis.

The angular size of the penumbra shadow at the observer's distance is given by the equation:

$$\psi = \eta + \theta \qquad (1)$$

The penumbra shadow angle θ can be calculated with the following equation:

$$\theta = \sin^{-1}\left(\frac{d_s + d_m}{R_{m-s}} \right) \qquad (2)$$

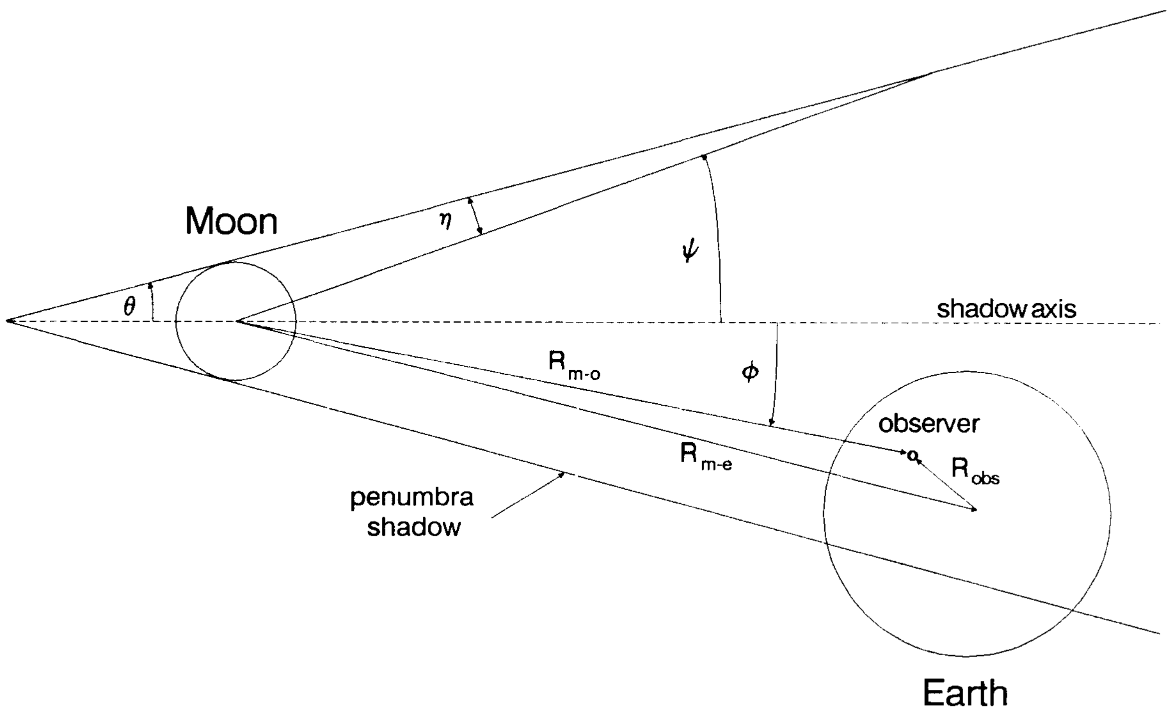
where

d_m = semidiameter of the moon

d_s = semidiameter of the sun

R_{m-s} = distance from the moon to the sun

Figure 1 - Penumbra Shadow Geometry



The angle η can be found with the equation

$$\eta = \sin^{-1}\left(\frac{d_m}{R_{m-o}}\right) \tag{3}$$

where R_{m-o} is *topocentric* distance from the moon to the observer.

The vector R_{obs} is the *geocentric* inertial position vector of the observer. The calculation of this vector was described in the FUNDAMENTAL ASTRONOMY column of the Fall 1989 issue.

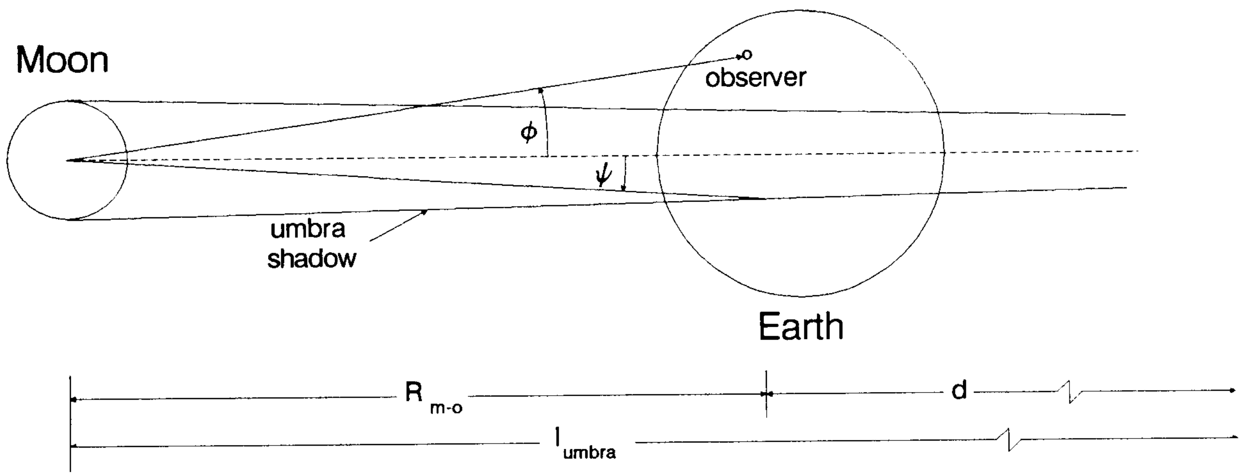
The relative geometry between the umbra shadow and the earth observer is shown in Figure 2. For this case, the shadow angle is given by:

$$\psi = \eta - \theta \tag{4}$$

The umbra shadow angle θ is calculated with

$$\theta = \sin^{-1}\left(\frac{d_s - d_m}{R_{m-s}}\right) \tag{5}$$

Figure 2 - Umbra Shadow Geometry



For the umbra shadow case, there are two possible situations. The type of eclipse is a function of the moon-to-observer distance and the length of the umbra shadow.

If $R_{m-o} > l_u \Rightarrow$ *annular eclipse* $\Rightarrow d = R_{m-o} - l_u$ (6a)

If $R_{m-o} < l_u \Rightarrow$ *total eclipse* $\Rightarrow d = l_u - R_{m-o}$ (6b)

where l_u is the selenocentric length of the umbra shadow and d is the length of the umbra shadow cone relative to an earth observer.

The umbra shadow length is calculated with the equation:

$$l_u = \frac{d_m R_{m-s}}{d_s - d_m} \tag{7}$$

The umbra angle at the observer's distance can now be found from

$$\psi = \sin^{-1}\left(\frac{d \sin\theta}{R_{m-o}}\right) \tag{8}$$

The separation angle between the shadow axis and the observer at any time during an umbra or penumbra eclipse is:

$$\phi = \cos^{-1}\left(\hat{U}_{m-o} \cdot \hat{U}_{m-s}\right) \tag{9}$$

The geometry of obscuration during a solar eclipse is described on pages 246-247 of Chapter 9 of the *Explanatory Supplement to the Ephemeris* (1974). The obscuration S can be computed from the following equation:

$$S = s^2 A + B - s \sin C \tag{10}$$

where

$$s = \frac{s_m}{s_s}$$

$$s_m = \sin^{-1}\left(\frac{0.272488}{R_{m-o} / R_e}\right) = \text{lunar semidiameter}$$

$$s_s = \frac{959.''63}{R_{s-o} / A_u} = \text{solar semidiameter}$$

$$A = \pi - (B + C)$$

$$B = \cos^{-1}\left(\frac{(1 + s - w)^2 + 1 - s^2}{2(1 + s - w)}\right)$$

$$C = \cos^{-1}\left(\frac{(1 + s^2) - (1 + s - w)^2}{2s}\right)$$

$$w = 1 + s - \cos^{-1}\left(\hat{U}_{m-o} \cdot \hat{U}_{s-o}\right) / s_s$$

R_e = equatorial radius of the earth (6378.14 kilometers)

R_{s-o} = distance from the sun to the observer

A_u = Astronomical Unit (149597870.66 kilometers)

$\hat{U}_{m-o}$ = unit pointing vector from the moon to the observer

$\hat{U}_{s-o}$ = unit pointing vector from the sun to the observer

The eclipse magnitude during any time of a solar eclipse is calculated from the equation

$$M = 2\ w \tag{11}$$

The following example illustrates output from SECLIPSE.BAS. This printout displays the local circumstances of the partial solar eclipse relative to Denver, Colorado on July 11, 1991.

Enter Penumbra Shadow

Calendar date	July 11, 1991
Local civil time	10 hours 43.70 minutes
Observer latitude	39° 45' 00" North
Observer longitude	104° 58' 48" West
Julian Date	2448449.240
Local sidereal time	6 hours 0.33 minutes
Lunar azimuth angle	130.637 degrees
Lunar elevation angle	65.485 degrees
Solar azimuth angle	129.448 degrees
Solar elevation angle	65.258 degrees
Lunar semidiameter	16 minutes 58.63 seconds
Solar semidiameter	15 minutes 43.91 seconds

Maximum Eclipse Conditions

Calendar date	July 11, 1991
Local civil time	11 hours 50.30 minutes
Observer latitude	39° 45' 00" North
Observer longitude	104° 58' 48" West
Julian Date	2448449.286
Local sidereal time	7 hours 7.11 minutes
Lunar azimuth angle	169.062 degrees
Lunar elevation angle	71.800 degrees
Solar azimuth angle	168.596 degrees
Solar elevation angle	72.059 degrees
Lunar semidiameter	16 minutes 59.27 seconds
Solar semidiameter	15 minutes 43.91 seconds
Magnitude	0.474
Obscuration	37.00 %

Exit Penumbra Shadow

Calendar date	**July 11, 1991**
Local civil time	**12 hours 56.45 minutes**
Observer latitude	**39° 45' 00" North**
Observer longitude	**104° 58' 48" West**
Julian Date	**2448449.332**
Local sidereal time	**8 hours 13.43 minutes**
Lunar azimuth angle	**213.972 degrees**
Lunar elevation angle	**69.030 degrees**
Solar azimuth angle	**215.345 degrees**
Solar elevation angle	**69.272 degrees**
Lunar semidiameter	**16 minutes 58.88 seconds**
Solar semidiameter	**15 minutes 43.92 seconds**

Program Notes

The program will begin by prompting the user for the geographic location of the earth observer. This information includes the latitude, *west* longitude, and altitude in meters. The software will also ask for the observer's time zone and the status of Daylight Savings Time. Please note the sign conventions and ranges of valid input for program SECLIPSE.

The software will also prompt the user for the initial calendar date at which to begin the search for solar eclipses. Please note that all fours digits of the calendar date must be input. The valid range of calendar dates is 1600 to 2200.

The last prompt displayed by the software will be

Please enter the search duration in days

The response to this should be the total number of days to search for solar eclipses.

Although the logic of SECLIPSE checks for all minima of the separation angle, it will only print the local circumstances for solar eclipses that are actually visible at the observer's location. This logic insures that the topocentric elevation angle is positive during the solar eclipse.

The lunar and solar ephemeris used in SECLIPSE.BAS is derived from "Low-Precision Planetary Formulae". All time calculations are based on Julian Ephemeris Time (JET). The conversion from JET to local civil time is the method of Morrison and Stephenson.

Program SECLIPSE.BAS *brackets* the times of lunar shadow entry and exit with the following simple QuickBASIC subroutine:

```
SUB BROOT (X1, X2) STATIC

    ' Bracket function root subroutine

    CALL OFUNCTION(X1, F1)

    DO
      X2 = X2 + FACTOR * (X2 - X1)
      CALL OFUNCTION(X2, F2)
    LOOP UNTIL F1 * F2 < 0#

END SUB
```

Upon entry to this subroutine, the variable X1 is the Julian Ephemeris Time of the separation angle minima and X2 = X1 ± Δt. A negative sign in the equation for X2 will cause the algorithm to search backwards in time and a positive sign makes the computer code search forward. The *objective* function or separation angle function is evaluated each time the line of code *CALL OFUNCTION* is executed. The subroutine *loops* through the objective function looking for a change in *sign* of the function. The constant *FACTOR* helps *accelerate* the search by expanding the interval each time through the loop. Program SECLIPSE.BAS uses values of Δt = 0.0007 days and FACTOR = 0.25.

The source code for finding a *root* of the objective function looks something like:

```
T2 = TMIN - .0007#
CALL BROOT(TMIN, T2)
CALL ROOT(TMIN, T2, JDATE)
```

where TMIN is the Julian Ephemeris Time of the separation angle minima. The root is returned in the QuickBASIC variable *JDATE*. Note that the JET of *maximum* eclipse is equal to *TMIN*.

The output from the software includes the azimuth and elevation (or altitude) angles of the sun and moon. Please note that these angles and the lunar and solar semidiameters are computed and displayed in the *topocentric* coordinate system.